Comparison of both calculation methods for cross wind vibration
in the Eurocode 1 EN 1991-1-4

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1. Introduction

In the past there were many publications about the cross wind vibration. But the number of the publications, which are applicable for the practical use, is limited. Starting with the proposal by Vickery [1], Scruton [2], Langer [3] and Petersen [4] and refinements by Ruscheweyh [5, 15, 19] a practicable calculation directive was introduced. This procedure is successfully used for more than 25 years and is part of actual standards [6, 7, 8]. Aside another procedure was developed by Vickery / Basu [10] with a numerical analysis by Daly [11]. This procedure was simplified by van Koten [12] and introduced in the CICIND Model Code [13]. A similar calculation model was established in the Eurocode 1 EN 1991-1-4 [8] as "procedure 2". Hence there are two different calculation procedures in the Eurocode, which can be used alternatively. Unfortunately the results differ significantly and lead to the question why these procedures are not harmonized. This article compares the both procedures and estimates both critically.

2. Procedure 1

The procedure 1 in the Eurocode is presented in numerous publications. These publications clarified also the background. Some examples are in [14] to [19]. Therefore the derivation should not be repeated. The calculation of the vibration amplitude is defined by equation (1):

$$\frac{y_{F,\max}}{b} = \frac{1}{St^2} \cdot \frac{1}{Sc} \cdot K \cdot K_W \cdot c_{lat} \tag{1}$$

given in the EN 1991-1-4 [8]. The mentioned parameters Scruton number, Sc, and the coefficient for the mode of vibration, K, are mechanical values and based on the structure. The other parameters, Strouhal number, St, the effective correlation length factor, K_W, and the lateral force coefficient, c_{lat}, are aerodynamical values and therefore dependent on the flow. The effective correlation length factor, K_W, catches the synchronisation of vortex shedding with the vibration motion. With increasing vibration amplitude this parameter increases also and shows transition from the external force excitation to a pseudo self-excitation. While the Strouhal number is mostly constant over a large wind speed range, the lateral force coefficient depends on many influence parameters, which is extreme evident e.g. for the Reynolds number dependency at cylinders. The correct value of the parameters St and c_{lat} have to be determined in experiments.

The procedure 1 shows its advantages with its basis on a simple comprehensible aerodynamic lateral force and the applicability on all structures and even for higher vibration modes. Consequently the procedure 1 gives a calculation for the cross wind vibration of chimneys, towers, masts, billboards, antennas, bridges, wires, bridge suspender, truss members, chemical towers, even slender high-rise buildings and other vibration susceptible buildings and construction elements. These information lead to predictions of the fatigue strength of the respective structures.

Figure 1 shows different examples of use of the cross wind vibration calculation according to procedure 1. The aerodynamic excitation force acts always at the part of the structure with the largest distortion of the respective vibration mode.

The most frequent use of cross wind vibration calculation can be found for chimneys and slender towers. Figure 2a shows a comparison of calculated and measured vibration amplitudes at the top of steel chimneys. The amplitude is presented versus the Scruton number and the conformity of measured and calculated values is obvious. Commonly the calculated values are slightly larger than the measured values (for each criticality of the air flow: sub-, super- resp. trans-critical) and show that the calculation is apparently on the safe side. Figure 2b shows an example of a bridge suspender [9]. Even at this example the conformity is unambiguously.

Figure 2a presents also a measuring curve which was created by Vickery / Watkins in 1963 [1]. This curve correspondence with the calculation according to procedure 1 as well as with the measured values from model experiments by Ruscheweyh and measurements on original chimneys. Please note that the amplitude increases steady with decreasing Scruton number and does not show any limitation at small Scruton numbers. This will be engrossed in the discussion about procedure 2.

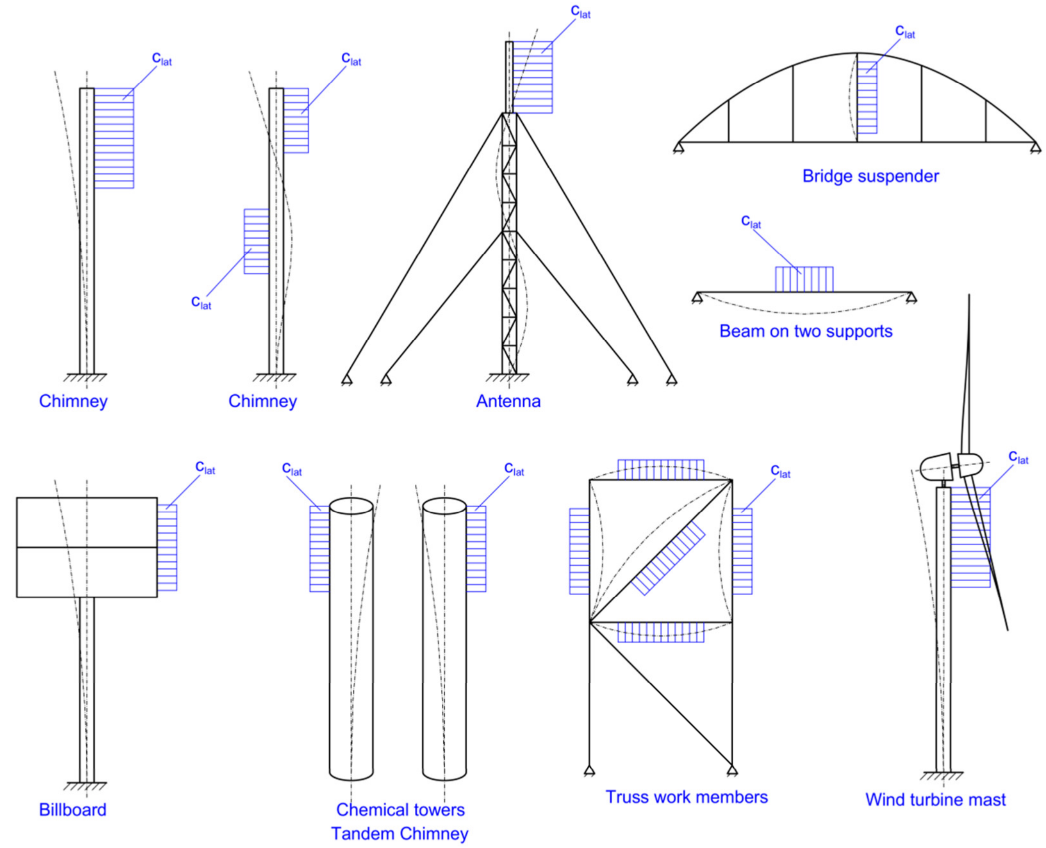


Fig. 1: Examples of use for the cross wind vibration calculation according to procedure 1

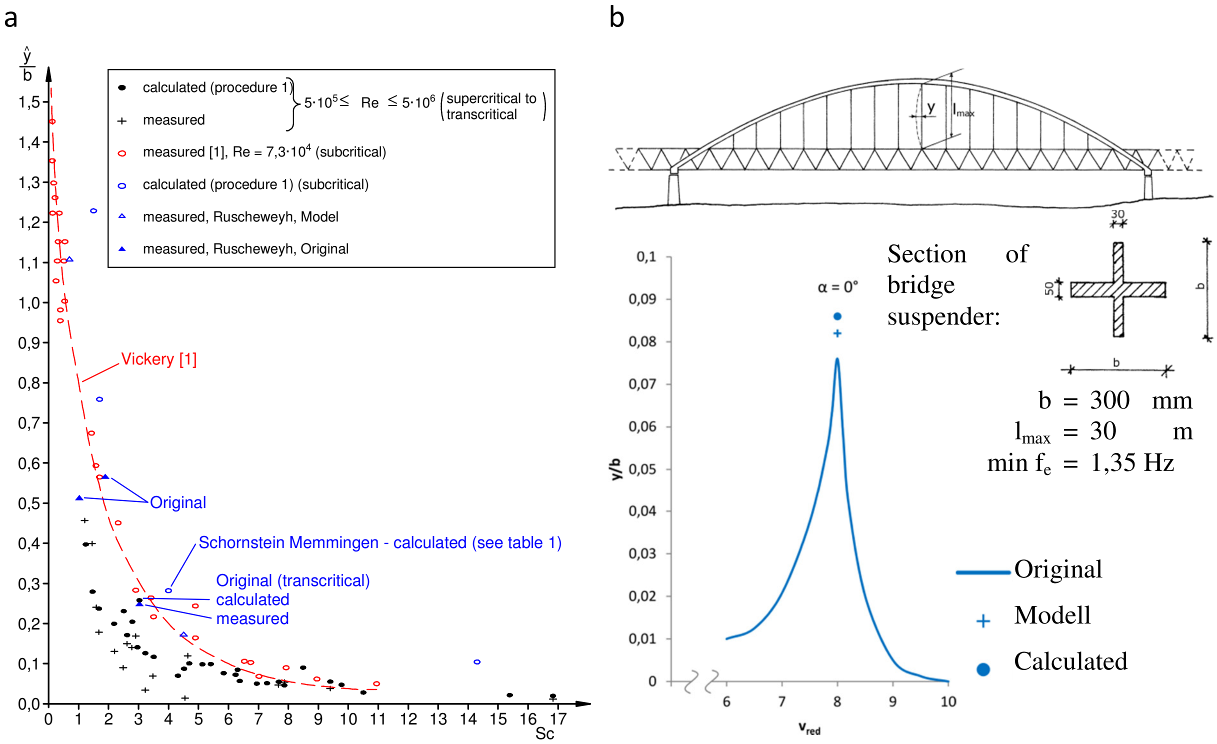


Fig. 2: a: Measured and calculated (Procedure 1) cross wind vibration ŷ/b versus the Scruton number Sc and model measurements by Vickery/Watkins [1] and Ruscheweyh [18]

b: Calculated and measured cross wind vibration amplitude at an original bridge suspender [9]

3. Procedure 2

The procedure 2 bases on the works of B. J. Vickery and R. Basu [10]. It emanates from a stochastic assumption of the vortex shedding and is related to characteristics of the vibration of concrete chimneys. The aerodynamic excitation force is introduced as “aerodynamic damping parameter”, K_a, and represents an approach by C. Scruton. Scruton hoped to find a simple calculation procedure, which is based on this aerodynamic damping. Unfortunately the experiments showed that this value is not constant and depends on many parameters, especially on the vibration amplitude. A simple solution could not be found with this approach. In a personal dialog with the author C. Scruton stated that he didn’t trace this approach any more. He solved the phenomenon of the cross wind vibration amplitude with graphical presentations of the aerodynamic and the structural damping.

Anyway the approach with the aerodynamic damping was picked up by several authors to generate a calculation procedure. The equation by Vickery is basis for the calculation of the aerodynamic coefficients by Daly [11]. The basic equation is:

$$\frac{y}{d}=\frac{\frac{gC_L\psi(h)}{8\pi^2 St^2}\cdot\frac{\rho d^2}{m_o}\cdot\left[\frac{\sqrt{\pi}l}{2(\lambda+2)}\right]^{\frac{1}{2}}\cdot\Phi(B,k)}{\left[\frac{1}{h}\int_0^h\psi^2(z)dz\right]^{\frac{1}{2}}[\beta_s-\beta_a]^{\frac{1}{2}}} \tag{2}$$

Without consideration of each individual parameter the term $[\beta_s-\beta_a]^{\frac{1}{2}}$ is of particular interest. It’s the square root of the difference of the structural damping β_s and the aerodynamic damping β_a. The problem of this solution is obvious: If the value β_a is larger than β_s there is no solution of the equation.

The aerodynamic damping is defined by Daly [11] as follows:

$$\beta_a=\frac{\rho d^2}{m_0}K_a \tag{3}$$

with

$$K_a=K_{a_0}\left[1-\left(\frac{y}{y_L}\right)^2\right] \tag{4}$$

K_{ao} = negative aerodynamic damping parameter
y = rms of the vibration amplitude at the chimney top

y_L = limiting amplitude of the vibration, rms

Putatively the equation (4) shows the complex trend of the aerodynamic damping. Figure 3 shows measured examples of the aerodynamic damping against the normalized vibration amplitude η = y/b by L. R. Wooton [20]. The damping drops with increasing Reynolds number, at Re = 1,39 · 10^6 the type of the trend even changes significantly. The curve calculated by equation (4) for Re = 5 · 10^5 was included into this diagram. The conversion into the relevant dimensions for Fig. 3 was done by equation (5):

$$\frac{2m\delta_a}{\rho b^2} = 4\cdot\pi\cdot K_{ao}\left[1-\left(\frac{y}{y_L}\right)^2\right]; \qquad \frac{y}{y_L} = \frac{\eta}{a_L} = 2.5\cdot\eta \tag{5}$$

With K_{ao} = 0,5 and a_L = 0,4 (acc. EN 1991-1-4, Procedure 2)

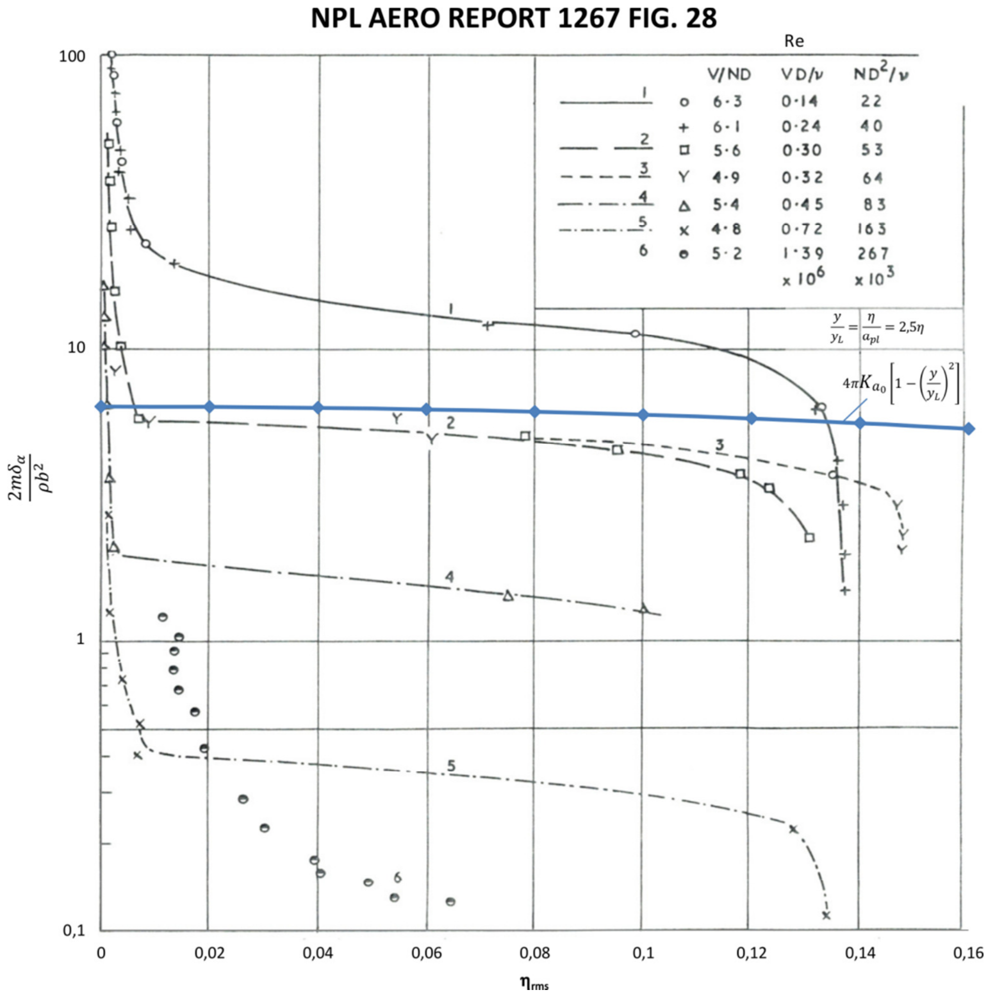
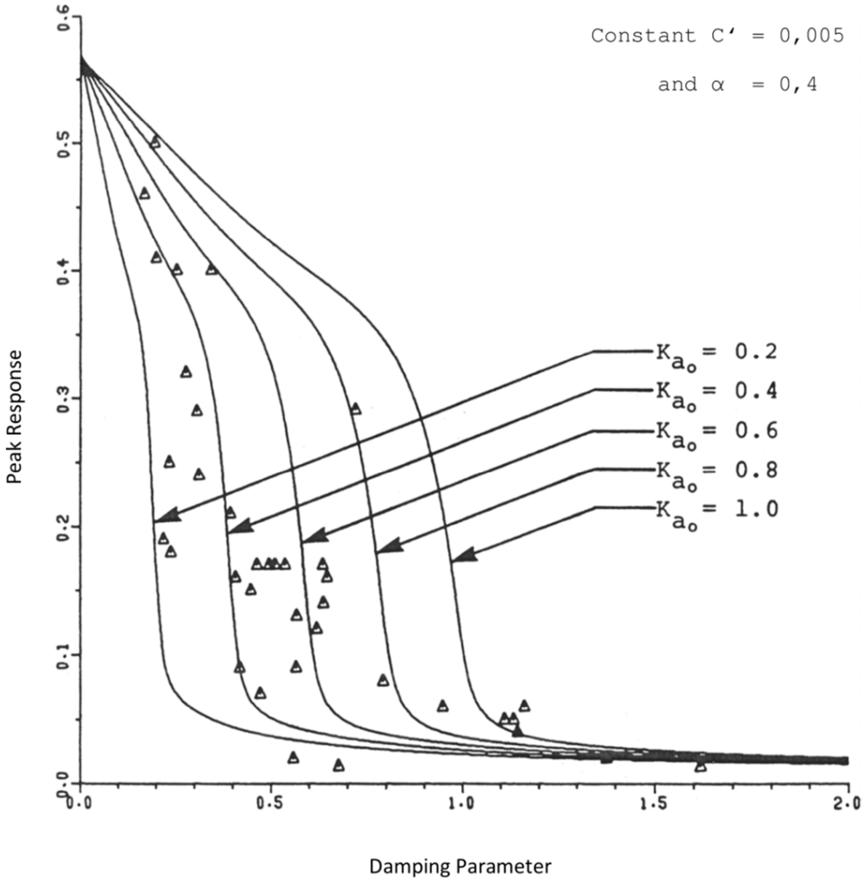


Fig. 3: Aerodynamic damping against normalized vibration amplitude η = y/b - measured values [20]. Comparison with the calculation procedure of equation (5)

This example shows only limited conformity between the measured values of curve 2 and 3. The curves 4 and 5, which catch the Reynolds number of $5 \cdot 10^5$ the best, miss the calculated line. The calculation procedure shows excessive large values with high amplitudes, especially where the measured values sink intensely. This means that this procedure shows large amplitudes for weak damped systems like e.g. steel chimneys and antenna masts. The rise with small η-values isn't also part of this calculation but does not have such a big influence on the fatigue calculation as the vibration amplitudes are very small.

The negative aerodynamic damping parameter, K_{ao}, must be measured in experiments. The original masts and chimneys have large Reynolds numbers which can't be attained in conventional wind tunnels. So this value must be measured at original structures.

Daly [11] analysed a list of 64 original chimneys, which was published by R. Pritchard [21]. The result was a large scatter band of the K_{ao} values. Daly has defined envelope curves and presented a calculation proposal. Fig. 4 shows an example for supercritical Reynolds numbers.



Predicted peak response for various values of the K_{ao} for supercritical flow

Fig. 4: Scatter range of the K_{ao} - values - analysis by Daly [11] at cylindrical chimneys in supercritical Reynolds number range.

The large scatter range prompted the author to check the degree of reliance of the list of Pritchard [21]. All stated literatures were obtained and the given information reviewed. The given data shape up as incomplete and partially wrongly interpreted:

- Pritchard deleted 22 chimneys from the list, as their data are not confident (personal letter by Prichard)
- Another 28 chimneys were part of groups or row alignments. For these cases the interference effect is significant. These measurements are not proper to scale the aerodynamic damping parameter.
- Three chimneys were equipped with aerodynamic measures. They are also inappropriate.
- At one chimney the decay curve of the damping measurement was interpreted as vortex shedding vibration.

After this correction the number of chimneys is reduced from primary 64 to 10, which have to be checked in more detail. For these chimneys two classes were established:

A: Chimneys with a full set of significant data as: natural frequency, measured structural damping, accurate known mass distribution, measured vibration amplitude.

B: Chimneys with incomplete set of data: e.g. natural frequency measured or calculated, damping estimated, mass distribution not accurate, vibration amplitude observed but not measured (From experience observation of vibration amplitudes are strongly over estimated.)

Result: There was no chimney to put into class A. In class B six chimneys had observed structure damping, the amplitude of one chimney was only observed. The mass distribution of the other three chimneys was estimated. Therefore a scaling of the K_{oa} value was not possible. However the analysis of Daly was adopted without any criticism and introduced in a calculation procedure. Because the K_{ao} values are set as much too high the calculated amplitudes are also very large.

In the Eurocode 1 this presentation by Vickery was introduced in a slightly altered shape as procedure 2. The equation is:

$$\frac{\sigma_y}{b} = \frac{1}{St^2} \cdot \frac{C_c}{\sqrt{\frac{Sc}{4 \cdot \pi} - K_a \cdot \left(1 - \left(\frac{\sigma_y}{b \cdot a_L}\right)^2\right)}} \cdot \sqrt{\frac{\rho \cdot b^2}{m_e}} \cdot \sqrt{\frac{b}{h}} \tag{6}$$

The maximum amplitude can be calculated with the peak factor k_p:

$$\max y = k_p \cdot \sigma_y \tag{7}$$

$$k_p = \sqrt{2 \cdot \left\{1 + \arctan\left(0{,}75 \cdot \frac{Sc}{4 \cdot \pi \cdot K_a}\right)^4\right\}} \quad (8)$$

In the equations (6) and (8) the aerodynamic damping parameter, K_a, and the normalised limited amplitude, a_L, are existent again. This limitation of the amplitude could not be confirmed in model experiments. On the contrary, the amplitude increases highly with decreasing Scruton number, as it can be seen in Fig. 2a. Hence the values K_{ao} and a_L meet not the reality and are therefore arbitrary. A calculation procedure based on arbitrary data cannot generate realistic values.

The damping of the structure is included in the Scruton number, so the square root under the fraction line also contains the difference of structural and aerodynamic damping. This leads to extreme large vibration amplitudes with small structural damping.

Equation (6) leads to a quadratic equation. The solution are given in the Eurocode as follows:

$$\left(\frac{\sigma_y}{b}\right)^2 = c_1 \pm \sqrt{c_1^{\,2} + c_2} \quad (9)$$

$$c_1 = \frac{a_L^{\,2}}{2} \cdot \left(1 - \frac{Sc}{4 \cdot \pi \cdot K_a}\right); \qquad c_2 = \frac{\rho \cdot b^2}{m_e} \cdot \frac{a_L^{\,2}}{K_a} \cdot \frac{C_c^{\,2}}{St^4} \cdot \frac{b}{h} \quad (10)$$

Calculation on different examples show much higher values with procedure 2 as with procedure 1, particularly in ranges which are out of every experience. Table 1 shows results calculated by equation (1) resp. (9) and (10).

Furthermore it has to be noted that the procedure 2 has no limitation of the critic wind speed. The use is also limited to cantilever systems and to the first natural frequency.

Chimney	Height	Width	structural damping	reduced mass	Scruton number	Strouhal number	natural frequency	critical wind speed	Reynolds number	lateral force coefficient	aerodynamic damping parameter	limited amplitude	Constant	Constant	Constant	peak factor	Amplitude according Procedure 1		Amplitude according Procedure 2		Comparison of Procedure 2 / 1
	h	b	δ_s	m_e	Sc	St	f_e	v_{crit}	Re	c_{lat}	K_a	a_L	C_C	c_1	c_2	k_p	y_1	y_1/b	y_2	y_2/b	y_2/y_1
	[m]	[m]	[-]	[kg/m]	[-]	[-]	[Hz]	[m/s]	[-]	[-]	[-]	[-]	[-]	[-]	[-]	[-]	[m]	[-]	[m]	[-]	[-]
Linz	36.9	1.22	0.020	255.5	5.49	0.180	0.83	5.60	4.552x10^{5}	0.292	0.587	0.4	0.034	0.02	7.3E-5	2.790	0.154	0.126	0.703	0.576	4.6
Essen (a/d = 5,7)	24.0	0.96	0.020	185.2	6.43	0.173	1.6	8.87	5.678x10^{5}	0.300	0.592	0.4	0.006	0.01	2.6E-6	3.025	0.115	0.120	0.429	0.446	3.7
Bomlitz	36.0	1.42	0.020	459.4	7.29	0.180	0.87	6.85	6.488x10^{5}	0.200	0.688	0.4	0.007	0.01	2.3E-6	2.976	0.092	0.065	0.670	0.472	7.3
B-Gouvy	35.0	1.42	0.020	445.5	7.07	0.180	0.93	7.36	6.968x10^{5}	0.200	0.739	0.4	0.007	0.02	2.6E-6	2.817	0.095	0.067	0.783	0.552	8.2
Ungarn (a/d = 4,8)	40.0	3.00	0.050	964.8	8.58	0.158	1.49	28.38	5.676x10^{6}	0.293	1.000	0.4	0.010	0.03	2.2E-5	2.706	0.315	0.105	1.837	0.612	5.8
Stendal	25.3	1.60	0.040	396.1	9.90	0.180	1.66	14.76	1.574x10^{6}	0.200	1.000	0.4	0.010	0.02	7.8E-6	2.864	0.077	0.048	0.847	0.529	11.1
Memmingen	55.0	1.42	0.014	362.2	4.02	0.180	0.5	3.97	3.759x10^{5}	0.479	1.000	0.4	0.032	0.05	2.9E-5	2.510	0.401	0.282	1.177	0.829	2.9
Ingelheim	27.0	1.02	0.014	204.8	4.44	0.180	1.24	6.97	4.719x10^{5}	0.200	0.554	0.4	0.034	0.03	7.7E-5	2.658	0.108	0.107	0.657	0.646	6.1
Dürrlauingen	30.0	1.90	0.020	565.6	5.01	0.180	1.74	18.41	2.331x10^{6}	0.200	1.000	0.4	0.010	0.05	7.7E-6	2.524	0.180	0.095	1.488	0.783	8.3
Lichterfelde	38.0	2.00	0.030	490.2	5.88	0.180	1.25	13.91	1.854x10^{6}	0.200	1.000	0.4	0.010	0.04	8.2E-6	2.546	0.161	0.081	1.486	0.743	9.2
Bruckmühl	30.0	1.60	0.028	350.3	6.13	0.180	1.68	14.92	1.591x10^{6}	0.200	1.000	0.4	0.010	0.04	7.4E-6	2.554	0.124	0.077	1.170	0.731	9.5

Table 1: Calculated vortex shedding cross wind vibration amplitude of 11 original steel chimneys according to procedure 1 resp. procedure 2 of EN 1991-1-4

4. Modification of the input data for procedure 1

The Strouhal number at a cylinder depends on different parameter; measured values of the Strouhal number scatter in a wide range. Verwiebe [24] found out, that the Strouhal number of 0.2, named in the DIN EN 4133:1991-11, leads to insecure calculation results. His finding based on original measurements - also at bridge suspenders - and on the analysis of cases of damage. This problem is valid especially at slender chimneys, which have a frequent appearing small critical wind speed. At bridge suspenders with cylindrical profile the largest amplitudes are measured at critical wind speeds corresponding with Strouhal numbers from 0.16 to 0.18. For the constructive design Verwiebe proposed to use a Strouhal number of 0.16 for conservative calculation. But he also gave cause for concern that the representative higher critical speed may lead to smaller load cycle numbers. In the Eurocode 1 and the DIN 1055-4 the Strouhal number for the calculation was named with 0.18 in 2005 instead of 0.2 in former standards.

A study of Clobes et al. [26] showed that with the procedure 1 the frequent cross wind vibration is calculated appropriately. For special dimensioning they also propose to use a Strouhal number of St = 0.16.

Certain weather situation, especially low temperatures and low turbulence in the wind flow, may lead to larger cross wind vibration amplitudes [24, 25]. These vibrations are part of the “low-cycle fatigue”. In order to cover this phenomenon Colbes et al. [26] propose to increase the K_W value to 0.95 (*Note: This large K_W-value is physically not reachable, as disturbances at the top of the chimney and near ground level prevent the regular vortex shedding*) and the maximum correlation length L/b to 30 for vibration large amplitudes y/b > 0.2 (*Note: Also this is a fictive value*). For very low temperatures the calculative air density should be enlarged to $\rho = 1.40\ kg/m^3$ and the cinematic viscosity should be reduced to $1.15 \times 10^{-5}\ m^2/s$.

This proposal has to be discussed, but only for the "low-cycle fatigue" case and as an additional check of the fatigue strength.

5. Summary and Conclusion

In a comparing study the calculation results of the both procedures 1 and 2 for vortex shedding cross wind vibrations in the Eurocode EN 1991-1-4 were valuated. The calculated vibration amplitudes differ up to the factor 11 comparing procedure 1 and 2. The results of procedure 2 are in every case larger than the results of procedure 1. The vibration amplitudes calculated by procedure 2 are often wide outside of the experiences. This discrepancy is caused firstly in the approach of the aerodynamic damping in the calculation method of procedure 2 and secondly by the overrated evaluation table of Pritchard. The procedure 1 provides reliable values for the "frequent occurring" vortex shedding vibration, the rare extreme values might be under estimated. These extreme values are part of the "low-cycle" fatigue and have to be handled separately. There are some considerations by Verwiebe and Clobes et al. to adapt some parameters of the procedure 1 for a more conservative calculation.

As the procedure 2 delivers too large dynamic loads, it has to be discussed to delete this procedure. But the surroundings of the Eurocode committee reports on the contrary trend; there are considerations to delete the procedure 1 which was used successfully in the civil engineering for over 25 years and didn't cause any complaints. On basis of the presented correlations and especially for the planning reliability for inventory structures this trend must be prohibited.

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